Fractural analysis of pre-cracked simply supported beam

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Annotation: the article considersbehaviour of a simply supported pre-cracked beam made of elasto-plastic material. The aim of the work is to study propagation of min-span fractures in the beam, defining of its behaviour in the local area and developing of techniques and methods to prevent further growth of a crack. There were applied vary verifymethods to analyse deflections, internal forces and stress intensity factor in the area of crack propagation.Analytical and numerical calculation had been used. For numerical solution ANSYS software is used, based on finite element method. According the solutions key features and conclusions are given.
Keywords: fracture, pre-cracked beam, stress intensity factor, load-bearing capacity, elasto-plastic material, finite element method.

The aim of the work is to study the load-bearing capacity of the pre-cracked beam with various lengths and widths of the crack. The beam made of elasto-plastic material and considered in 2-dimensional case.

There are three basic types of fractures. In the first group (it is the type I of the cracks) the fracture is originated from tension, in the second (type II.) from shear, and in the third from twisting (type III.) [1].

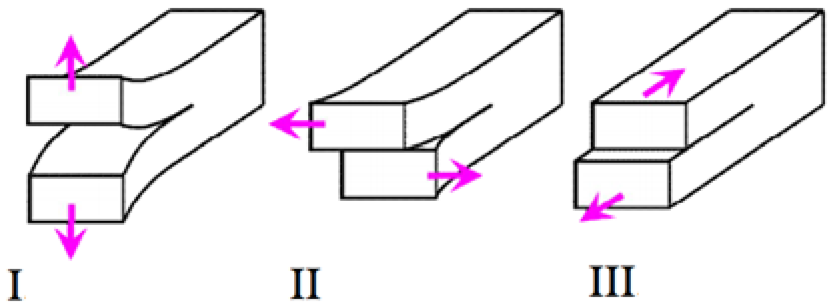

Fig. 1. – Basic types of fracture

Opening mode (I): the crack surfaces separate symmetrically with respect to the planes *XY* and *XZ*.

Sliding mode (II): the crack surfaces slide relative to each other symmetrically with respect to the plane *XY* and skew-symmetrically with respect to the plane *XZ*.

Tearing mode (III): the crack surfaces slide relative to each other skew-symmetrically with respect to both planes *XY* and *XZ*.

This study deals with only first group of fractures in a beam which cause by vertical load.

The geometry is made with SolidWorks. It was done for each case of the crack (size parameters as various lengths and widths ware varied) and after exported to ANSYS Workbench. The material properties and some parameters of the crack were defined with ANSYS Engineering data satellite. For each case of the input data stress intensity factor (K_I), deflections and stresses ware define for different geometry sizes and forms of the crack by ANSYS and also analytical solution [2,3].

For the analysis a 2D element model was built up with SolidWorks for each geometry case of the crack. The analysis was made in ANSYS.

The general parameters of the model show in table 1.

Table № 1

The general parameters of the beam

Length of the beam (S), mm	Height of the beam (W), mm	Thickness of the beam (B), mm	Depth of the crack (a), mm	Distribution load (q), kPa	Young's module (E), GPa
6000	800	400	50..150	1	210

The geometry was imported to ANSYS, there the boundary conditionals and load are applied according the Figure 2.

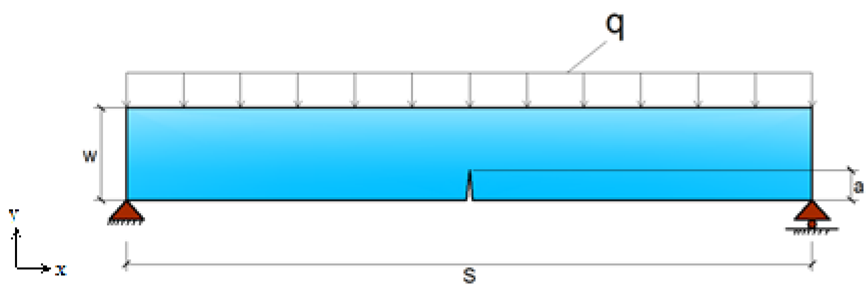

Fig. 2. – Principal scheme of the beam

The next step is mesh generating of the model. In the area of crack we have to use smaller side sizes of the elements [4,5]. We shall define optimal size of finite elements. To solve this problem we start from elements with 40 mm side size and we will compare the values of deflection with the previous one each step.

Table № 2

Maximum vertical displacements in the mid-span of the beam in depending on elements size

Element size, mm	Deflection, m	Difference, %
40	0,0657	
16	0,0658	0,24
8	0,0660	0,19
4	0,0660	0,06
2	0,0660	0,02

As can be seen from the previous table, for those sizes of elements there is no significant difference between deflections of the beam.

Also, we have to check influence of the element size for another parameter of the analysis - Stress intensity factor [6]. This relation is shown in the table.

Table № 3

Relation between stress intensity factor and element sizes

Element size , mm	Stress intensity factor (K_I), $MPa\sqrt{mm}$	Difference, %
40	1,5351	
16	3,2552	52,84
8	4,9875	34,73
4	6,0267	17,24
2	6,7728	11.02
1	7,2167	6,15
0.5	7,4681	3,37

It can be seen, that the difference of the Stress intensity factor with 1 mm and 0,5 mm elements size less than 5%, which is acceptable. For the current numerical experiment we have to select element with 0,5 mm side size for the crack area.

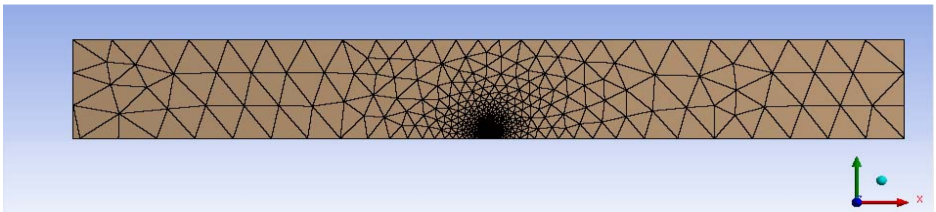

Fig. 3. – Finite element model of the beam

The following figure demonstrates the beam with mid-span crack (depth of the crack is $a = 80$ mm), where the value of stress intensity factor is equal to 0,58065.This value is close to the result comes from analytical solution for the same input data. [7].

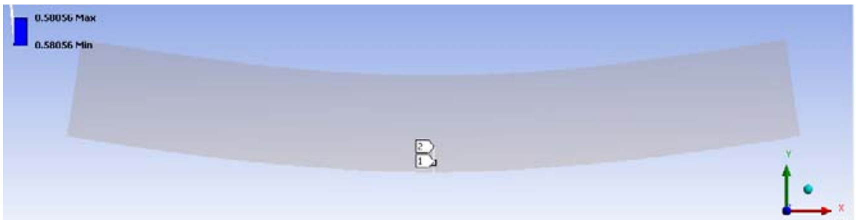

Fig. 4. – Stress intensity factor (K_I)

The shape of the crack is changed after applying the load. It can be seen from the figure 5.

a) b)

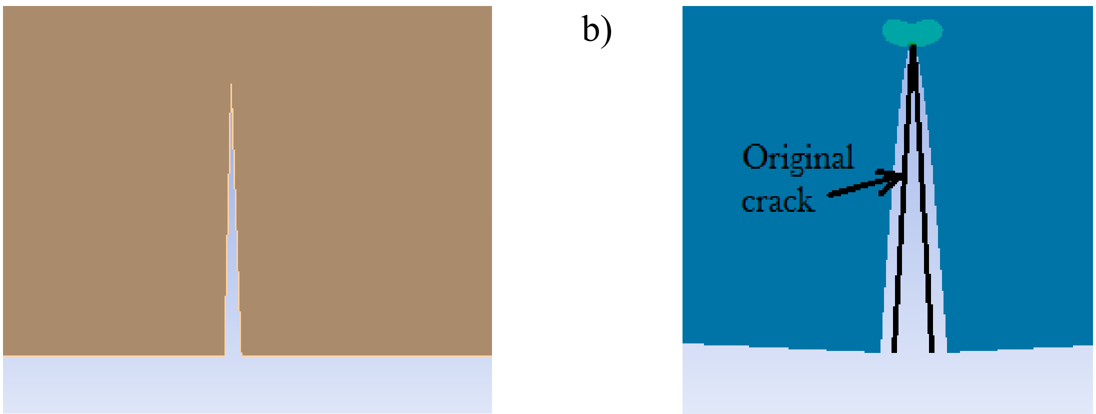

Fig. 5. – Shapes of the crack

a) original shape; b) shape with crack opening displacement

Figure 6 presents distribution of the normal stress at the top of the crack. There are plastic and elastic material behaviors in this area.

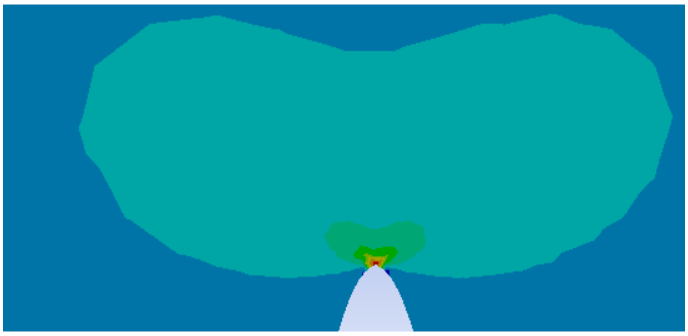

Fig. 6. – Normal (horizontal) stress distribution at the top of crack

Numerical calculation results of the displacement of the beam for the different values of the crack length and constant value of its deep show in table 4.

Table № 4

Maximum values of deflection for different length of the mid-span crack

Deep of the crack, mm	Length of the crack, mm	Vertical displacement, mm
150	3	18,47
150	4	18,48
150	5	18,47
150	6	18,46
150	7	18,46
150	8	18,46
150	10	18,48
150	15	18,47
150	20	18,46
150	25	18,48

The next step of the research is analysis of the K_I changing in depends on shape of the crack [8]. The new models have crack with rounded top and diameters of the rounding are 1 mm, 2 mm, 5 mm, and 10 mm. Geometry of the one of them is shown in Figure 7.

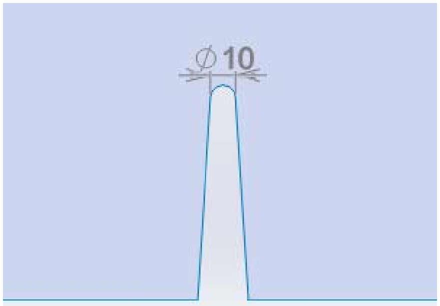

Fig. 7. – Geometry of the rounded crack with diameter 10 mm

These models of the beam were compared to the same one, but with sharp angle in the top of the crack (diameter of the rounding $D = 0$ mm). The results are shown in the following table.

Table № 5

Results for different geometry parameters of the crack

Load, kN/m	$D = 0$ mm		$D = 1$ mm		$D = 2$ mm		$D = 5$ mm		$D = 10$ mm	
	Def., *mm*	K_I, $MPa\sqrt{mm}$	Def., *mm*	K_I, $MPa\sqrt{mm}$	Def., *mm*	K_I, $MPa\sqrt{mm}$	Def., *mm*	K_I, $MPa\sqrt{mm}$	Def., *mm*	K_I, $MPa\sqrt{mm}$
8	22,06	2.787	22,35	0.237	22,37	0.070	22,39	0.088	22,47	0.058
16	44,12	5.573	44,71	0.475	44,75	0.139	44,77	0.178	44,95	0.116
24	66,18	8.360	67,06	0.712	67,12	0.209	67,16	0.266	67,42	0.175

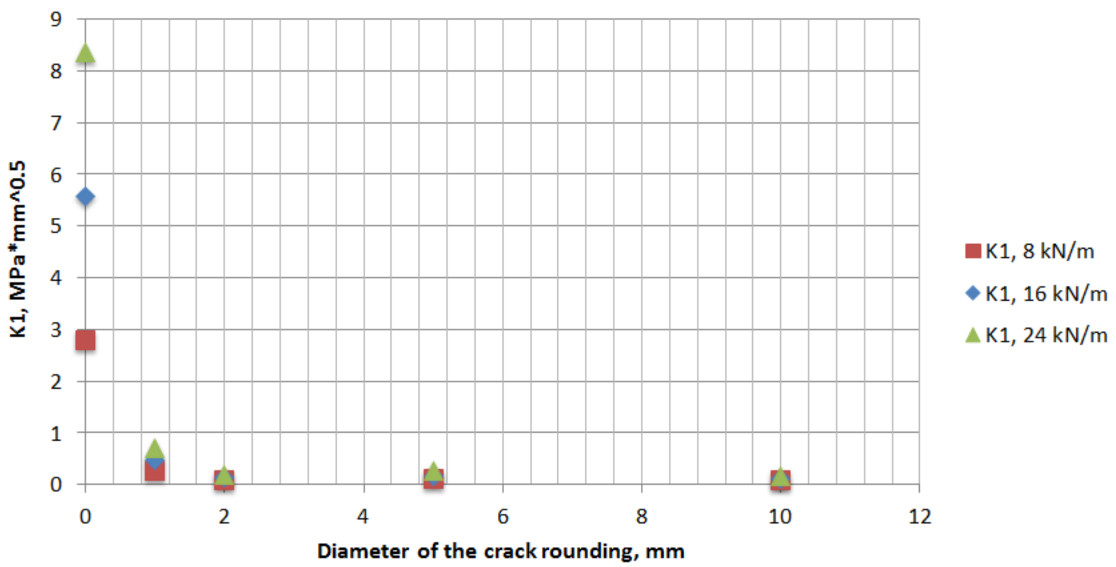

Fig. 8. – Plot of stress intensity factor values (K_I), depends on diameter of the

crack rounding and different values of the load

The plot (Fig. 8) shows the relation between stress intensity factor (K_I), diameter of the crack rounding and different values of the load.

This chart describes the relation between deflection of the beam, diameter of the crack rounding and different values of the load.

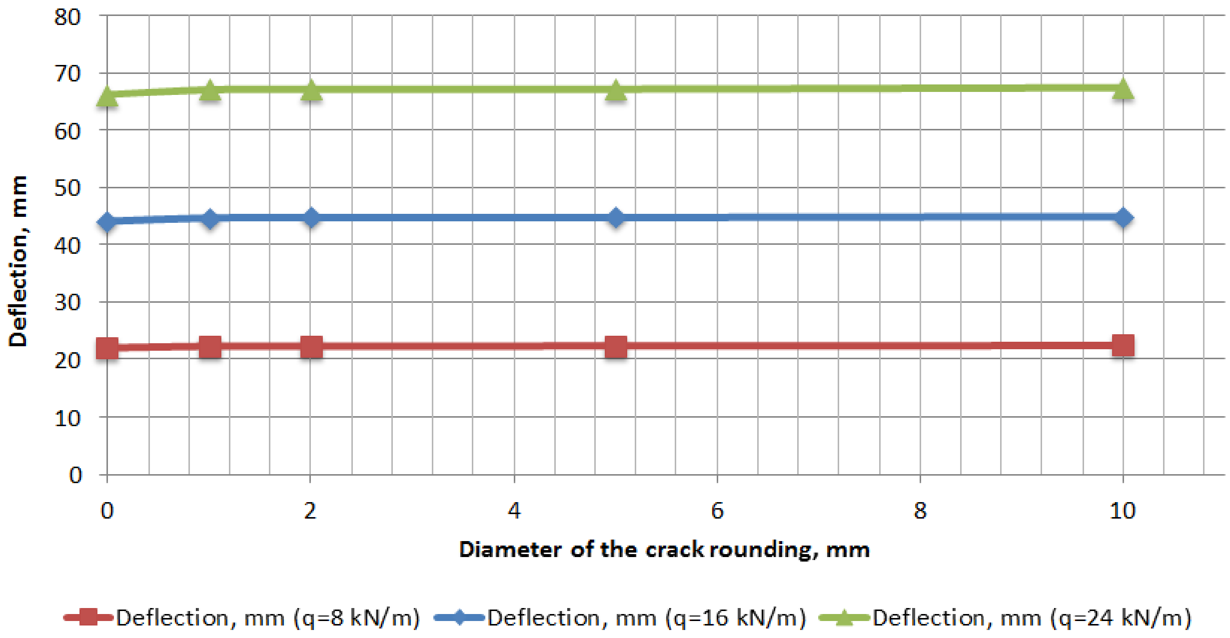

Fig. 9. – Relation between deflection of the beam, diameter of the crack rounding and different values of the load

As can be seen from the previous plots, the rounding in the top of crack may significantly decrease value of stresses,whereas the vertical displacements have almost the same values.

To verify numerical solution results we have to compare them with analytical solution results. Following formulas are used as analytical solution.

Let $x = a/w$.Where a – depth of the crack, w – height of the beam.

Stress Intensity Factor (K_I) can be obtain as $K_I = \sigma\sqrt{\pi a}Y$ [2],

where

$$Y = \left(\frac{3}{\sqrt{\pi}}\frac{S}{w}\right)\left[\frac{1{,}99 - x(1-x)(2{,}15 - 3{,}93x + 2{,}7x^2}{2(1+2x)(1-x)^{3/2}}\right],$$

To calculate deflection of the beam we use the formula

$$V_L = \frac{F}{E' \cdot B}\left(\frac{S}{w-a}\right)^2 [1{,}193 - 1{,}98\, a/w + 4{,}478(a/w)^2 - 4{,}443\,(a/w)^3 + 1{,}739(a/w)^4].$$

In case of the crack depth $a = 80$ mm,stress intensity factor $K_I = 0{,}5806$.

Table № 6

Analytical results for the deflection of the beam

Depth of the crack, mm	K_I, $MPa\sqrt{mm}$	Deflection, mm
50	0,3489	19,85
60	0,4392	20,06
70	0,5165	20,29
80	0,5831	20,55
90	0,6443	20,83
100	0,6986	21,14

Depth of the crack, mm	K_I, $MPa\sqrt{mm}$	Deflection, mm
110	0,7485	21,48
120	0,7948	21,86
130	0,8384	22,26
140	0,8799	22,70
150	0,9198	23,18

Table № 7

Analytical results of stress intensity factor for different depth of the crack

Depth of the crack, mm	Analytical solution		ANSYS		Difference in K_I, %	Difference in deflection, %
	K_I, $MPa\sqrt{mm}$	Deflection, mm	K_I, $MPa\sqrt{mm}$	Deflection, mm		
50	0.3489	1.985	0.3510	1.989	0.6	0.2
80	0.5841	2.055	0.5806	2.058	0.6	0.2
120	0.7948	2.186	0.8002	2.191	0.7	0.2
180	1.0340	2.483	1.0409	2.491	0.7	0.3

The table 7 is performed comparing analytical solution results and ANSYS results for stress intensity factor and deflection of the beam. It is clear from the data, that differences between two ways of analysis no more than 0,7%.

The following plots illustrate relations between stress Intensity factor (Figure 11) and deflections (Figure 12) of the beam and a/w ratio.

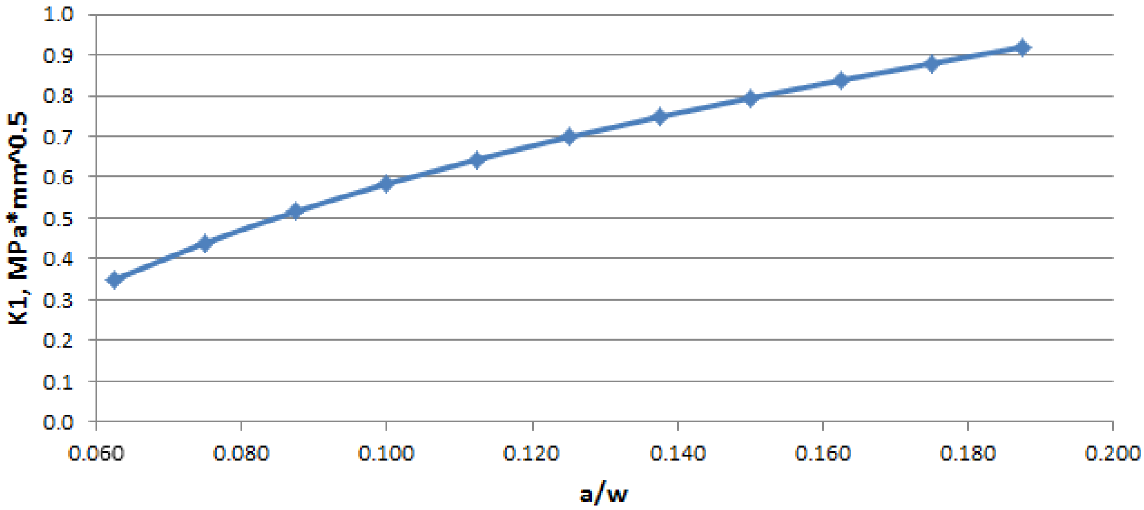

Fig. 11. – Variation of Stress Intensity Factor (K_I) with depth of the crack

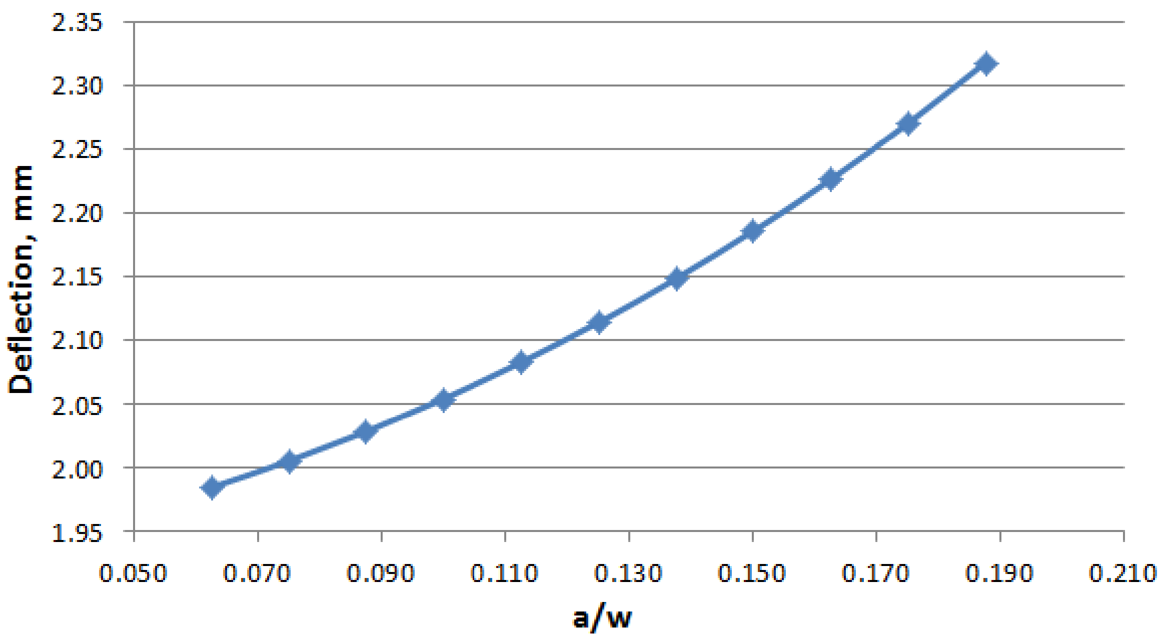

Fig. 12. – Variation of deflection with depth of the crack

The results of the analysis of the pre-cracked beam under different values of load are listed in Figure 13.

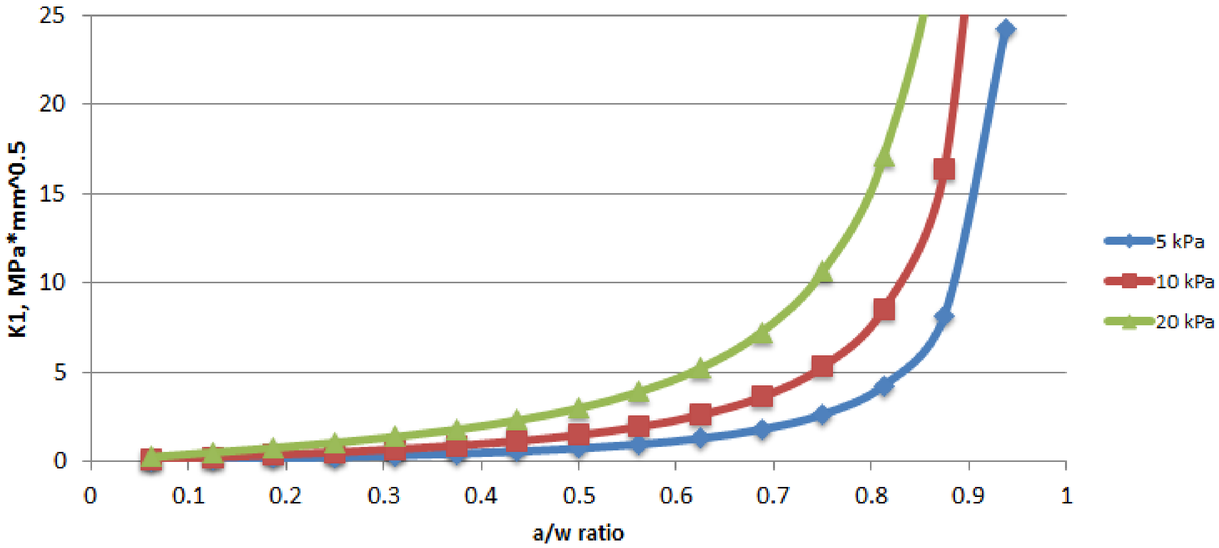

Fig. 13. – Plot of stress intensity factor vs. crack depth of the beam

According to the results, the relationship between the stress intensity factor at the crack tip and the crack height is plotted under different loading levels (5 kPa, 10 kPa, 20 kPa), which indicates that the stress intensity factor of the crack tip in the plain concrete beam increase sharply along with the rising of load and crack height (Figure 13).

According the results it can be obtained that the most important role for the load-bearing capacity of the beam plays deep of the crack and shape of its tip. In the case of rounded tip the value of the stresses may be significantly decreased. However, the width of the crack does not cause changing of stress strain state of the beam. It means that the drilling a hole in the top of the crack with diameters 1-5 mm can decrease stresses in the area of crack and prevent its further growth.

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